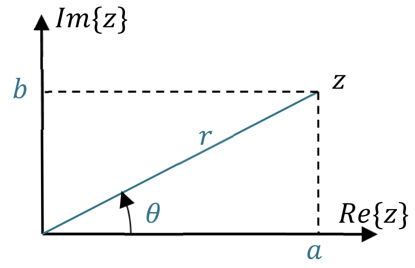
FÓRMULAS MATEMÁTICAS

NÚMEROS COMPLEJOS

Sea z un número complejo: $z \in \mathbb{C}$



Forma rectangular o cartesiana

$$z = a + jb$$

donde $j = \sqrt{-1}$, $a = Re\{z\}$ es la parte real de z y $b = Im\{z\}$ es la parte imaginaria de z

Forma polar

$$z = re^{j\theta}$$

donde $r = |z|$ es la magnitud de z y $\theta = \measuredangle z = arg\{z\}$ es la fase de z.

Fórmula de Euler

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Relación entre la representación polar y cartesiana de z:

$$a = r\cos\theta \qquad b = r\sin\theta$$

$$r = \sqrt{a^2 + b^2} \qquad \theta = arctan\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

Operaciones con complejos

- Si $z_1 = a_1 + jb_1 \quad z_2 = a_2 + jb_2$ entonces,

$$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$z_1 z_2 = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + b_1 a_2)$$

$$\frac{z_1}{z_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)} = \frac{(a_1 a_2 + b_1 b_2) + j(-a_1 b_2 + b_1 a_2)}{a_2^2 + b_2^2}$$

- Si $z_1 = r_1 e^{j\theta_1}$ $\quad z_2 = r_2 e^{j\theta_2}$ entonces,

$$z_1 z_2 = (r_1 r_2) e^{j(\theta_1+\theta_2)} \qquad \frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right) e^{j(\theta_1-\theta_2)}$$

Complejo conjugado

El complejo conjugado de z se escribe como z^* y viene dado por:

$$z^* = a - jb = re^{-j\theta}$$

Relaciones de interés:

$$zz^* = r^2 \qquad z - z^* = j2Im\{z\} \qquad (z_1 z_2)^* = z_1^* z_2^*$$

$$\frac{z}{z^*} = e^{j2\theta} \qquad (z_1 + z_2)^* = z_1^* + z_2^* \qquad \left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$$

$$z + z^* = 2Re\{z\}$$

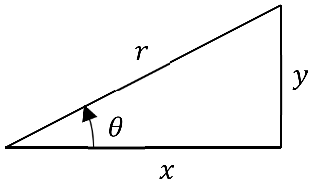
$$a = Re\{z\} = \frac{z + z^*}{2} \qquad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$b = Im\{z\} = \frac{z - z^*}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$z^n = r^n e^{jn\theta} = r^n(\cos(n\theta) + j\sin(n\theta))$$

$$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$$

Para el triángulo representado en la figura



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\cos^2\theta$$

Otras relaciones de interés:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$a\cos\alpha + b\sin\alpha = \sqrt{a^{2+}b^2}\cos\left(\alpha - tan^{-1}\frac{b}{a}\right)$$

SERIES GEOMÉTRICAS

$$\sum_{k=0}^{N} \alpha^k = \begin{cases} \dfrac{1-\alpha^{N+1}}{1-\alpha}, & \alpha \neq 1 \\ N+1, & \alpha = 1 \end{cases}$$

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=n}^{\infty} \alpha^k = \frac{\alpha^n}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=0}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \quad |\alpha| < 1$$

$$\sum_{k=0}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \quad |\alpha| < 1$$

EXPANSIÓN EN SERIES DE POTENCIAS

$$e^{\alpha} = \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = 1 + \alpha + \frac{1}{2!}\alpha^2 + \frac{1}{3!}\alpha^3 + \cdots$$

$$(1+\alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!}\alpha^2 + \cdots + \binom{n}{k}\alpha^k + \ldots + \alpha^n$$

$$ln(1+\alpha) = \alpha - \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 - \cdots + \frac{(-1)^{k+1}}{k}\alpha^k + \cdots, \quad |\alpha| < 1$$

FUNCIONES EXPONENCIALES Y LOGARÍTMICAS

$$e^{\alpha}e^{\beta} = e^{\alpha+\beta}$$

$$\frac{e^{\alpha}}{e^{\beta}} = e^{\alpha-\beta}$$

$$\ln(\alpha\beta) = \ln\alpha + \ln\beta$$

$$\ln\frac{\alpha}{\beta} = \ln\alpha - \ln\beta$$

$$\ln\alpha^{\beta} = \beta\ln\alpha$$

$$\log_b N = \log_a N \log_b a = \frac{\log_a N}{\log_a b}$$

ALGUNAS INTEGRALES DEFINIDAS

$$\int_a^b x^n dx = \frac{1}{n+1} x^{n+1}|_a^b$$

$$\int_a^b e^{cx} dx = \frac{1}{c} e^{cx}|_a^b$$

$$\int_a^b x e^{cx} dx = \frac{1}{c^2} e^{cx}(cx-1)|_a^b$$

$$\int_a^b \cos(cx)\, dx = \frac{1}{c}\sin(cx)|_a^b$$

$$\int_a^b \sin(cx)\, dx = -\frac{1}{c}\cos(cx)|_a^b$$

$$\int_a^b x\cos(cx)\, dx = \frac{1}{c^2}(\cos(cx) + cx\sin(cx))|_a^b$$

$$\int_a^b x\sin(cx)\, dx = \frac{1}{c^2}(\sin(cx) - cx\cos(cx))|_a^b$$

$$\int_a^b e^{gx}\cos(cx)\, dx = \frac{e^{gx}}{g^2+c^2}(g\cos(cx) + c\sin(cx))|_a^b$$

$$\int_a^b e^{gx}\sin(cx)\, dx = \frac{e^{gx}}{g^2+c^2}(g\sin(cx) - c\cos(cx))|_a^b$$

PULSO GAUSSIANO

$$\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx = \sigma\sqrt{2\pi}\,, \quad \sigma > 0$$

$$\int_{-\infty}^{\infty} x^2 e^{-x^2/2\sigma^2} dx = \sigma^3\sqrt{2\pi}\,, \quad \sigma > 0$$

FUNCIÓN SINC

$$sinc(t) = \frac{\sin(\pi t)}{\pi t}$$